American University of Beirut
Department of Computer Science
CMPS 211 – Discrete Mathematics – Fall 14/15

Please solve the following exercises and submit **BEFORE 12:00 pm (noon) of Tuesday 9th, December.**

Exercise 1 (10 points)

Give a recursive algorithm for computing nx whenever n is a positive integer and x is an integer, using just addition.

Exercise 2 (15 points)

Give a recursive algorithm for finding the sum of the first n positive integers and prove that it is correct.

Exercise 3 (10 points)

Give a recursive algorithm for finding the maximum of a finite set of integers, making use of the fact that the maximum of n integers is the larger of the last integer in the list and the maximum of the first n – 1 integers in the list.

Exercise 4 (10 points)

Give a recursive algorithm for finding a mode of a list of integers. (A mode is an element in the list that occurs at least as often as every other element.)

Exercise 5 (15 points)

Describe a recursive algorithm for multiplying two non- negative integers x and y based on the fact that $xy = 2(x \cdot (y/2))$ when y is even and $xy = 2(x \cdot \lfloor y/2 \rfloor)+x$ when y is odd, together with the initial condition $xy = 0$ when $y = 0$. Prove that your algorithm is correct.

Exercise 6 (10 points)

Devise a recursive algorithm to find a^{2^n}, where a is a real number and n is a positive integer. [*Hint:* Use the equality $a^{2^{n+1}} = (a^{2^n})^2$.]

Exercise 7 (10 points)

Devise a recursive algorithm to find the nth term of the sequence defined by $a_0 = 1$, $a_1 = 2$, $a_2 = 3$, and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$, for $n = 3,4,5,....$

Exercise 8 (10 points)

Use merge sort to sort 4, 3, 2, 5, 1, 8, 7, 6 into increasing order. Show all the steps used by the algorithm.

Exercise 9 (10 points)

Prove that the merge sort algorithm given in the lecture is correct.